Market simulation survey

White paper

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Abstract

Market simulation is employed in solving various financial problems, from risk management to business case evaluation. We review the applications of this method, their requirements and popular simulation methods.

1. Introduction

Many financial problems require that a portfolio of financial contracts is evaluated under the market conditions at certain points in the future. Although we cannot predict the future market conditions, we can model the probability distribution of the market variables, such as interest rates, currency exchange rates, stock and commodity prices. From those, we can calculate the probability distribution of the portfolio's value.

A common technique for computing the portfolio value distribution is market simulation. We start with a random draw of a market scenario: for each relevant market variable (also known as risk factor) we draw a sample realisation of its evolution from today's value to the given moment in the future. The market scenario is used as input for pricing the portfolio. Thus, one draw of a market scenario gives one realisation of the future value of the portfolio. By repeating the process, we obtain an estimate of the portfolio's value distribution.

We use the objective (real-world) probability measure to draw market scenarios. By contrast, the fair price of the portfolio is calculated as expected value under a pricing measure (e.g. risk-neutral measure), conditional on the drawn scenario. The result of the simulation is the portfolio value distribution under the objective measure.

2. Possible applications

2.1 Counterparty credit risk assessment

The risk of losses due to a counterparty default is often measured by the potential future exposure [3]. It is used for trade acceptance, determining the credit charge, as input to economic capital calculations, and for other purposes.

PFE calculation for derivative contracts (such as off-balance sheet trades) requires the distribution function of the contracts' values at several points in the future, including fairly long time horizons (several years).

2.2 Value at risk calculation

Value at risk [4] and expected shortfall (sometimes referred to as conditional value at risk) show the portfolio's sensitivity to market moves on a short term (typically 10 days or shorter). These measures are often used by financial organisations for monitoring their martket risks. Both value at risk and expected shortfall are easily to calculate from the portfolio value distribution at the given time horizon.

2.3 Project evaluation

In corporate management, the decision to start a new project is based on a business case evaluation. The evaluation reveals if the project's added value to the corporation will be greater than its expenses. One can evaluate a project by posing a question: "If this was someone else's project, would I invest in it?" Finding the answer often requires an insight into the future behaviour of the market.

2.4 Hedging performance assessment

When a new pricing model or calibration method is verified, testing the quality of the model-based hedge is of vital importance. Observing the hedge behaviour on the simulated market scenarios allows one to judge the new model's performance and compare it to the rivals.

2.5 Investment strategy evaluation

In asset and liability management, competing asset allocation decisions can be compared by their risk and return forecasts. Such forecasts can be obtained from the portfolio value distribution under simulated market conditions.

2.6 Trading algorithm test

Before a trading algorithm is taken into production, it is subjected to rigorous tests on real-world market evolutions. Simulated market scenarios provide a wealth of test cases for such algorithms.

3. Requirements

The above applications impose the following requirements on the simulation.

Long time horizon. The market must be simulated for long time periods (many years).

Preserving statistical properties. The model must faithfully reproduce essential statistical features of the historical time series for the risk factors. These features, also referred to as stylised facts, include heteroskedasticity (differing variance) with volatility clustering [8], autocorrelation [9], excess kurtosis (fat tails) [7] and mean reversion [10]. Not surprisingly, stylised facts are often specific to asset class: for instance, time series of commodity prices exhibit different behaviour than interest rate time series.

Path consistency. Simulated market scenarios must be suitable for pricing path-dependent products. The price of a path-dependent product depends on the values of risk factors *on and before* the valuation point. For such products, the simulated path of each risk factor must not only produce the correct terminal distribution, but also correct joint distributions of the risk factor's values sampled at any number of points in the path.

Capturing dependencies. Different market variables (stock and commodity prices, interest and exchange rates) are interrelated; their time series reveal statistical dependence. The models for risk factors have to behave accordingly.

Accounting for portfolio effects. Portfolio valuation routine must handle aggregation effects (e.g. the cases when different trades partially compensate each other). If netting or collateralisation agreements are in place, credit risk calculations must take them into account.

Conditioning on default. In PFE calculations it often appears that the mark-to-market value of the portfolio is statistically dependent on the creditworthiness of the counterparty. Examples of such cases are discussed in [6] and [11]. Therefore, the simulation must be able to model the market conditional on the counterparty default at a given point in the future.

Backtesting. The outputs of the simulation must be tested against the historical time series for the risk factors [12]. Backtesting of the risk models is often a regulatory requirement.

4. Simulation process

The complete process of market simulation can be broken down into the following steps.

1. **Define the goals.** Decide which statistics need to be calculated.
2. **Build the portfolo.** Specify the financial contracts and their netting and collateralisation structure.
3. **Identify risk factors.** Find the risk factors relevant to the portfolio and choose models for them.
4. **Set the valuation date.** Determine the future date for which the statistics will be calculated. For counterparty risk computations, this is the date of counterparty default.
5. **Find intermediate dates.** Produce the list of dates (between today's date and the valuation date) when risk factors need to be simulated for pricing of path-dependent contracts in the portfolio.
6. **Generate scenarios.** Draw random numbers, calculate risk factor values and the value of the portfolio.
7. **Compute statistics.** Use the generated distribution of portfolio values to produce the required statistical measures.

5. Scenario generation

The approaches to market simulation can be divided into three categories: parametric, non-parametric and semi-parametric.

5.1 Non-parametric simulation

The non-parametric, or historical, simulation uses randomly drawn returns from the historical time series to build the evolution paths of the risk factors. Let $X = \{x_i\}$ denote the set of fisk factors relevant to the portfolio in question.

5.1.1 Method

Suppose we have the historical data for each risk factor, $x_i(t)$, from the moment P somewhere in the past up to the moment T ("today"), which will be the starting point of our simulation. We would like to simulate the evolution of $\{x_i\}$ to the moment F in the future; $P < T < F$.

We begin with choosing the granularity δt. Then, we calculate series of returns Δx_i^k, where

$$\Delta x_i^k = x_i(t_k) - x_i(t_{k-1})$$

$$t_k = P + \delta t * k$$

$$k = 1 \dots \frac{T-P}{\delta t}$$

Commonly, $\delta t = 1\ day$, so Δx_i^k are daily returns. The method, however, remains the same for any δt, be it a week or an hour. Without loss of generality, we can consider $x_i(t)$ to be the logarithm of the risk factor's value, thus making Δx_i^k log-returns, or choose to work with absolute returns for some asset classes and log-returns for others.

Let

$$M = \frac{T-P}{\delta t}$$

$$N = \frac{F-T}{\delta t}$$

Both M and N are rounded down to the nearest integer, if needed. Vector $\{x_i\}$ at time F is given by

$$\{x_i\}(F) = \{x_i\}(T) + \sum_{j=1}^{N} \{\Delta x_i^{r_j}\}$$

where r_j is a random value drawn from the uniform distribution $[1 \dots M]$. For instance, if $\delta t = 1\ day$, we randomly select N days from history and for each risk factor add the returns realised on those days to today's value.

5.1.2 Benefits

Historical simulation has the following benefits.

- Captures statistical dependencies between risk factors.
- Provides path-consistent scenarios.
- Reproduces some stylised facts (e.g. fat tails of the daily return distribution).
- Is easy to implement.

5.1.3 Drawbacks

The shortcomings of the historical simulation are as follows.

- No conditioning on counterparty default.
- Fails to reproduce some stylised facts (e.g. serial autocorrelation of returns).
- May produce unrealistically looking yield curves or negative forward rates.
- Fixed time step δt may lead to big computational expenses at long time horizons.

5.2 Parametric simulation

Parametric (or model-based) simulation builds scenarios according to mathematical models of risk factors.

5.2.1 Method

Model-based simulation can be broken down into four steps.

1. Choose a suitable model for each risk factor. The risk factors of the same asset class typically share a model; only model parameters differ for individual risk factors. A simple example of a model is the geometric Brownian motion, which assumes that the evolution of a risk factor's value X is given by the equation

$$dX = \mu X dt + \sigma X dW$$

where W is a Brownian motion. Here, μ and σ are parameters of the model.

2. Calibrate the model. For each risk factor, the model parameters are chosen to achieve the best possible fit to the historical time series. In the example of geometric Brownian motion, the parameters can be computed by maximum likelihood estimation.
3. Set up correlation. The correlation algorithm depends on the models chosen for risk factors. Correlation parameters, again, are calculated for the best approximation of the joint history of risk factors.
4. Perform simulation. A number of random variables is drawn for each scenario (the exact number depends on the amount of risk factors and the model choice). The random values are fed to the model, yielding one realisation of the market.

5.2.2 Benefits

Model-based simulation has the following strong points, if the choice of models is right.

- Can generate scenarios conditional on counterparty default.
- Captures statistical dependencies between risk factors.
- Provides path-consistent scenarios.
- Reproduces some stylised facts.
- Not limited to a fixed time step.
- Creates realistic yield curves.

5.2.3 Drawbacks

The weakness of the model-based simulations is the requirement to have an adequate model for each risk factor. It is difficult (if not impossible) to find models that realise all of the above benefits. One often has to make trade-offs, for example, between getting path-consistent scenarios and capturing some of the stylised facts. The multitude of models for risk factors makes it difficult to correlate them.

5.3 Semi-parametric simulation

Semi-parametric simulation algorithms are combinations of parametric and non-parametric algorithms. A semi-parametric approach would combine some of the benefits of the other two groups, and suffer from some of the problems pertinent to both groups. The exact combination of properties depends on the construction of such algorithm.

6. Links to literature

A survey of stochastic processes used in market simulation is presented in [2]; another set of models is discussed in [1]. Some approaches to simulating interest rates are reviewed in [9]; the paper also presents a semi-parametric simulation algorithm. Modelling of interest rate curves is further explored in [5]. The model adopted by commercial software vendor RiskMetrics is documented in [13].

7. Conclusion

Market simulation can deliver an insight into the future behaviour of financial markets. In the multitude of market simulation techniques, there is no single solution that fits all problems. There is, however, a set of well-developed methods that provide satisfactory results in various special cases. Market simulation remains an active research subject.

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